PHYSICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 89

Total Marks : 27

Max. Time : 30 min.

Topics : Elasticity and Plasticity

Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.1 to Q.5	(3 marks, 3 min.)	[15,15]
Subjective Questions ('–1' negative marking) Q.6	(4 marks, 5 min.)	[4, 5]
Match the Following (no negative marking) (2 × 4)	(8 marks, 10 min.)	[8, 10]

COMPREHENSION

ELASTICITY AND PLASTICITY

The property of a material body by virtue of which it regains its original configuration (i.e. shape and size) when the external deforming force is removed is called elasticity. The property of the material body by virtue of which it does not regain its original configuration when the external force is removed is called plasticity.

Deforming force : An external force applied to a body which changes its size or shape or both is called deforming force.

Perfectly Elastic body : A body is said to be perfectly elastic if it completely regains its original form when the deforming force is removed. Since no material can regain completely its original form so the concept of perfectly elastic body is only an ideal concept. A quartz fiber is the nearest approach to the perfectly elastic body.

Perfectly Plastic body : A body is said to be perfectly plastic if it does not regain its original form even slightly when the deforming force is removed. Since every material partially regain its original form on the removal of deforming force, so the concept of perfectly plastic body is only an ideal concept. Paraffin wax, wet clay are the nearest approach to a perfectly plastic bodies.

Cause of Elasticity : In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. These forces are known as intermolecular forces. When no deforming force is applied on the body, each molecule of the solid (i.e. body) is in its equilibrium position and the inter molecular forces between the molecules of the solid are maximum.

On applying the deforming force on the body, the molecules either come closer or go far apart from each other. As a result of this, the molecules are displaced from their equilibrium position. In other words, intermolecular forces get changed and restoring forces are developed on the molecules. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium positions and hence the solid (or the body) regains its original form.

STRESS

When deforming force is applied on the body then the equal restoring force in opposite direction is developed inside the body. The restoring forces per unit area of the body is called stress.

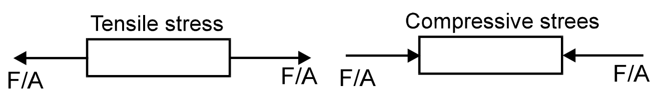
$$\text{stress} = \frac{\text{restoring force}}{\text{Area of the body}} = \frac{F}{A}$$

The unit of stress is N/m^2 or Nm^{-2}. There are three types of stress

1. **Longitudinal or Normal stress**

When object is one dimensional then force acting per unit area is called longitudinal stress.

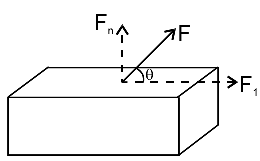
It is of two types : (a) compressive stress (b) tensile stress



Examples :

(i) Consider a block of solid as shown in figure. Let a force F be applied to the face which has area A. Resolve $\vec{F}$ into two components :

$F_n = F \sin\theta$ called normal force and $F_t = F \cos\theta$ called tangential force.



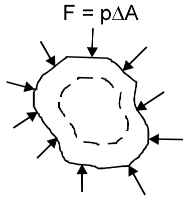
$$\therefore \ \text{Normal (tensile) stress} = \frac{F_n}{A} = \frac{F\sin\theta}{A}$$

2. **Tangential or shear stress**

It is defined as the restoring force acting per unit area tangential to the surface of the body. Refer to shown in figure above

Tangential (shear) stress = $\frac{F_t}{A} = \frac{F\cos\theta}{A}$

The effect of stress is to produce distortion or a change in size, volume and shape (i.e. configuration of the body).



STRAIN

The ratio of the change in configuration (i.e. shape, length or volume) to the original configuration of the body is called strain

i.e. Strain, $\in = \frac{\text{change in configuration}}{\text{original configuration}}$

It has no unit

(i) Longitudinal strain : This type of strain is produced when the deforming force causes a change in length of the body. It is defined as the ratio of the change in length to the original length of the body.

Consider a wire of length L **:** When the wire is stretched by a force F, then let the change in length of the wire is ΔL shown in the figure.

$\therefore$ Longitudinal strain , $\in_\ell = \frac{\text{change in length}}{\text{original length}}$ or Longitudinal strain = $\frac{\Delta L}{L}$

HOOKE'S LAW AND MODULUS OF ELASTICITY

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain

or stress = constant × strain or $\frac{\text{stress}}{\text{strain}}$ = Modulus of Elasticity.

This constant is called modulus of elasticity.

Thus, modulus of elasticity is defined as the ratio of the stress to the strain.

Modulus of elasticity depends on the nature of the material of the body and is independent of its dimensions (i.e. length, volume etc.).

Unit : The SI unit of modulus of elasticity is Nm^{-2} or Pascal (Pa).

1. **Young's modulus of elasticity**

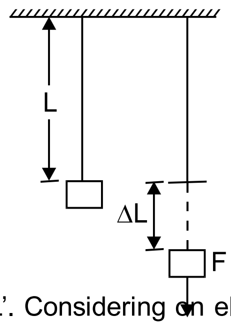
It is defined as the ratio of the normal stress to the longitudinal strain.

i.e. Young's modulus (Y) $= \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

Normal stress = F/A,

Longitudinal strain = ΔL/L

$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$ $\therefore \Delta\ell = \frac{F\ell}{AY}$

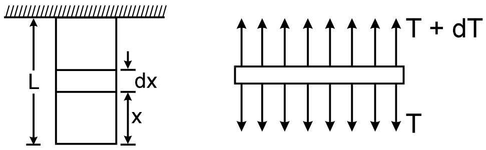


ELONGATION OF ROD UNDER IT'S SELF WEIGHT

Let rod is having self weight 'W', area of cross-section 'A" and length 'L'. Considering an element at a distance 'x' from bottom.

then $T = \frac{W}{L}x$

elongation in 'dx' element = $\frac{T.dx}{Ay}$



Total elongation $s = \int_0^L \frac{T\,dx}{Ay} = \int_0^L \frac{W\,x\,dy}{LAy} = \frac{WL}{2Ay}$

Note : One can do directly by considering total weight at C.M. and using effective length $\ell/2$.

Illus. 1. One end of a wire 2 m long and 0.2 m^2 in cross-section is fixed in a ceiling and a load of 4.8 kg is attached to the free end. Find the extension of the wire. Young's modulus of steel = 2.0×10^{11} N/m^2. Take g = 10 m/s^2.

Sol. We have

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{\ell/L}$$

with symbols having their usual meanings. The extension is

$$\ell = \frac{TL}{AY}$$

As the load is in equilibrium after the extension, the tension in the wire is equal to the weight of the load

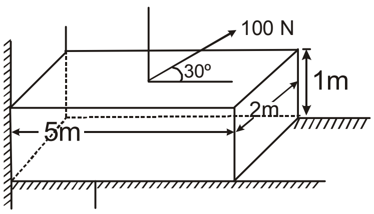
= 4.8 kg × 10 m/s^2 = 48 N.

Thus, $\ell = \frac{(48\text{ N})(2\text{m})}{(0.2\times10^{-4}\text{m}^2)\times(2.0\times10^{11}\text{N/m}^2)}$

$= 2.4 \times 10^{-5}$ m.

Illus. 2.

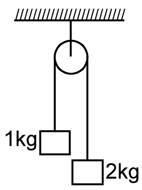
Find out longitudinal stress and tangential stress on a fixed block shown in figure when a tangential force of 100 N magnitude is applied on the block.



Sol. Longitudinal or normal stress $\Rightarrow$ $\sigma_l = \frac{100 \sin 30°}{5\times2} = 5$ N/m^2

Tangential stress $\Rightarrow$ $\sigma_t = \frac{100 \cos 30°}{5\times2} = 5\sqrt{3}\text{N/m}^2$

Illus. 3. Two blocks of masses 1 kg and 2 kg are connected by a metal wire going over a smooth pulley as shown in figure. The breaking stress of the metal is 2×10^9 N/m^2. What should be the minimum radius of the wire used if it is not to break? Take g = 10 m/s^2



Sol. The stress in the wire = $\frac{\text{Tension}}{\text{Area of cross-section}}$. To avoid breaking, this stress should not exceed the breaking stress.

Let the tension in the wire be T. The equations of motion of the two blocks are,

T – 10 N = (1 kg) a

and 20 N – T = (2 kg) a.

Eliminating a from these equations,

T = (40/3) N.

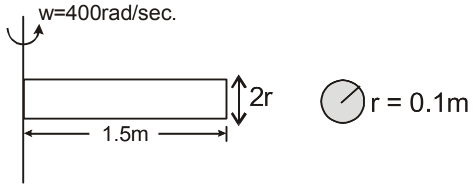
The stress = $\frac{(40/3)\text{N}}{\pi r^2}$.

If the minimum radius needed to avoid breaking is r,

$$2 \times 10^9 \frac{N}{m^2} = \frac{(40/3)\,N}{\pi r^2}.$$

Solving this, $r = 4.6 \times 10^{-5}$ m.

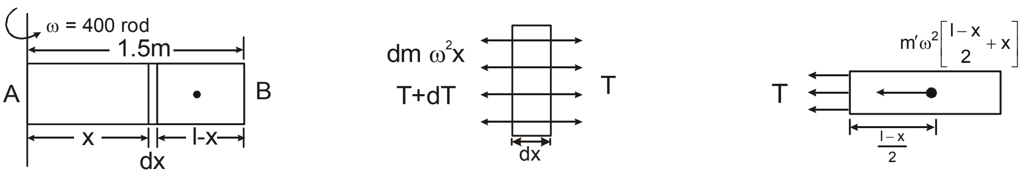
Illus. 4. A rod of 1.5 m length and uniform density 10^4 kg/m³ is rotating at an angular velocity 400 rad/sec. about its one end in a horizontal plane. Find out elongation in rod.
Given $y = 2 \times 10^{11}$ N/m²



Sol. mass of shaded portion

$$m' = \frac{m}{\ell}(\ell - x) \qquad \text{[where m = total mass = } \rho A \ell \text{]}$$

$$T = m'\omega^2\left[\frac{\ell - x}{2} + x\right] \qquad \Rightarrow T = \frac{m}{\ell}(\ell - x)\,\omega^2\left(\frac{\ell + x}{2}\right) \qquad T = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$



this tension will be maximum at A $\left(\frac{m\omega^2\ell}{2}\right)$ and minimum at 'B' (zero), elongation in element of width 'dx' = $\frac{Tdx}{Ay}$

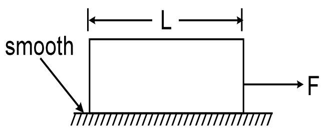
Total elengation

$$\delta = \int \frac{Tdx}{Ay} = \int_0^{\ell} \frac{m\omega^2(\ell^2 - x^2)}{2\ell\, Ay}\,dx$$

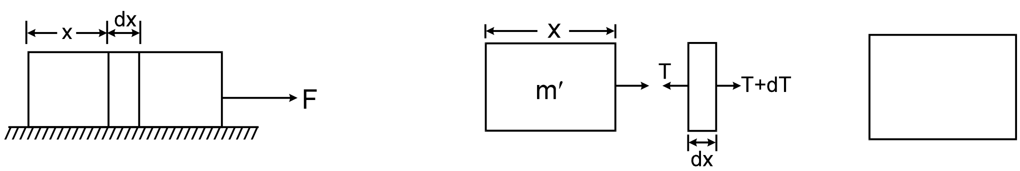
$$\delta = \frac{m\omega^2}{2\ell Ay}\left[\ell^2 x - \frac{x^3}{3}\right]_0^{\ell} = \frac{m\omega^2 \times 2\ell^3}{2\ell Ay \times 3} = \frac{m\omega^2\ell^2}{3Ay} = \frac{\rho A\ell\omega^2\ell^2}{3Ay}$$

$$\delta = \frac{\rho\omega^2\ell^3}{3y} = \frac{10^4 \times (400) \times (1.5)^3}{3 \times 2 \times 10^{11}} = 9 \times 10^{-3}\ \text{m} = 9\text{mm}$$

Illus. 5. Find out the elongation in block. If mass, area of cross-section and young modulus of block are m, A and y respectively.



Sol.



Acceleration, $a = \frac{F}{m}$ then $T = m'a$ where $\Rightarrow$ $m' = \frac{m}{\ell}x$

$$T = \frac{m}{\ell}x\,\frac{F}{m} = \frac{Fx}{\ell}$$

Elongation in element 'dx' = $\frac{Tdx}{Ay}$

total elongation, $\delta = \int_0^{\ell} \frac{T dx}{Ay} \quad d = \int_0^{\ell} \frac{F x dx}{A \ell y} = \frac{F\ell}{2Ay}$

Note : -

In this problem, if friction is given between block and surface (μ = friction coefficient), and

Case : (I) $F < \mu mg$

(II) $F > \mu mg$

Then in both cases answer will be total elongation $\delta = \frac{F\ell}{2Ay}$

Now answer the follwing :

1. A wire alongates by 1.0 mm when a load W is hanged from it. If this wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be

(A) 0.5 m (B) 1.0 mm (C) 2.0 mm (D) 4.0 mm

2. The length of a metal wire is ℓ_1 when the tension in it is T_1 and is ℓ_2 when the tension is T_2. The natural length of the wire is

(A) $\frac{\ell_1 + \ell_2}{2}$ (B) $\sqrt{\ell_1 \ell_2}$ (C) $\frac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$ (D) $\frac{\ell_1 T_2 + \ell_2 T_1}{T_2 + T_1}$

3. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break

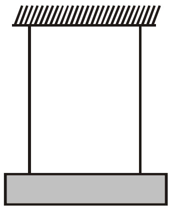
(A) when the mass is at the highest point

(B) when the mass is at the lowest point

(C) when the wire is horizontal

(D) at an angle of $\cos^{-1}(1/3)$ from the upward vertical

4. Two wires of equal length and cross-section area suspended as shown in figure. Thier Young's modulus are Y_1 and Y_2 respectively. The equivalent Young's modulus will be

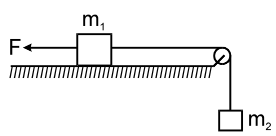


(A) $Y_1 + Y_2$ (B) $\frac{Y_1 + Y_2}{2}$ (C) $\frac{Y_1 Y_2}{Y_1 + Y_2}$ (D) $\sqrt{Y_1 Y_2}$

5. A steel wire and a copper wire of equal length and equal cross-sectional area are joined end to end and the combination is subjected to a tension. Find the ratio of (a) the stresses developed in the two wires and (b) the strains developed. Y of steel = 2×10^{11} N/m². Y of copper = 1.3×10^{11} N/m².

6. A steel rod of cross-sectional area 4 cm² and length 2 m shrinks by 0.1 cm as the temperature decreases in night. If the rod is clamped at both ends during the day hours, find the tension developed in it during night hours. Young's modulus of steel = 1.9×10^{11} N/m².

7. Consider the situation shown in figure. The force F is equal to the $m_2 g/2$. If the area of cross-section of the string is A and its Young's modulus Y, find the strain developed in it. The string is light and there is no friction anywhere.



Answers Key

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1. (B) **2.** (C) **3.** (B) **4.** (B)

5. (a) 1 (b) $\frac{\text{strain in copper wire}}{\text{strain in steel wire}} = \frac{20}{13}$

6. 3.8×10^4 N **7.** $\frac{m_2 g(2m_1 + m_2)}{2AY(m_1 + m_2)}$